Discrete FT of a sinusoid

The power spectrum of a sinusoid ***A sin (2πν_{sine}t_k+φ)***:

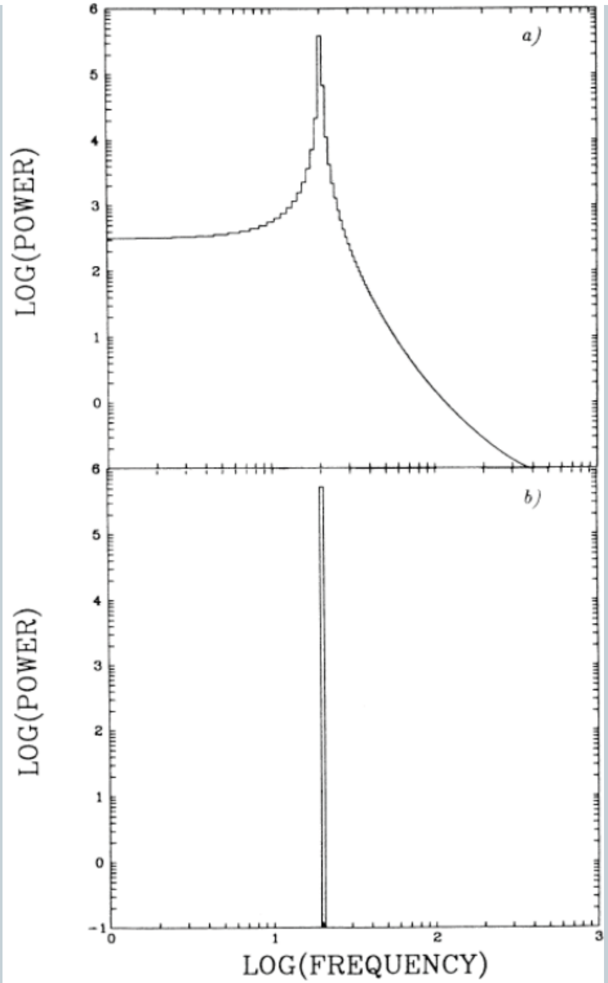
$$|a_j|^2 = \frac{1}{4}A^2N^2\left(\frac{\sin \pi x}{\pi x}\right)^2 \left[\left(\frac{\pi x/N}{\sin \pi x/N}\right)^2 + \left(\frac{\pi x/N}{\sin[\pi(2j+x)/N]}\right)^2 + \right.$$

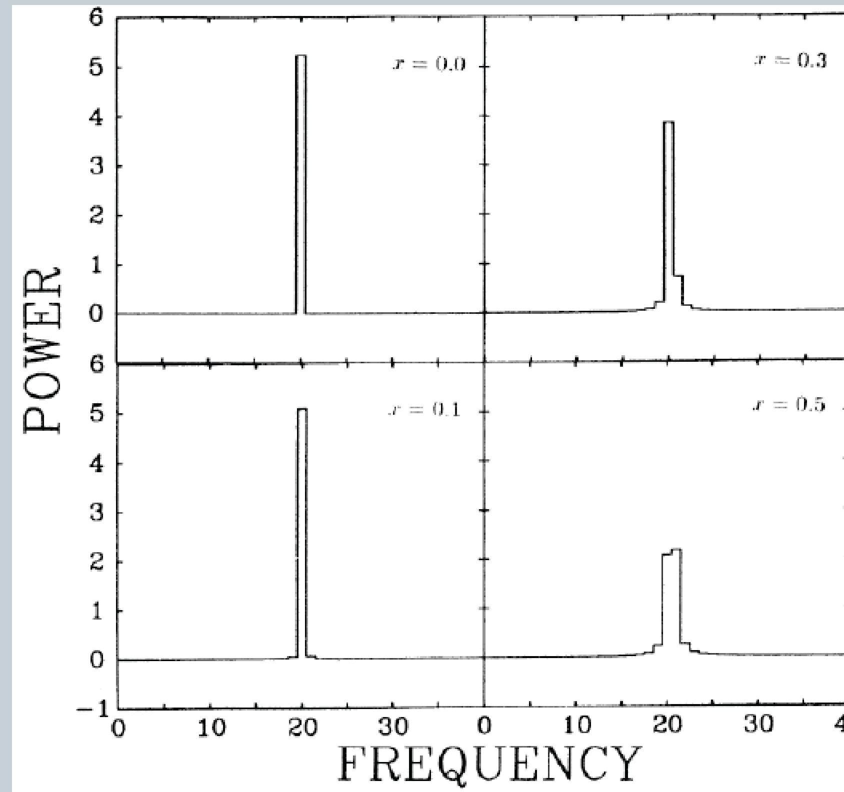
$$\left. + 2\left(\frac{\pi x/N}{\sin \pi x/N}\right)\left(\frac{\pi x/N}{\sin[\pi(2j+x)/N]}\right)\cos[(N-1)(2\pi(j+x)/N)+2\phi]\right]$$

$$x = (\nu_{\mathrm{sine}} - \nu_j)T$$

$$\approx \frac{1}{4}A^2N^2\left(\frac{\sin \pi x}{\pi x}\right)^2 \qquad x/N \ll 1 \text{ and } 0 \ll j/N \ll \frac{1}{2}$$

Discrete FT of a sinusoid





Periodogram & Power Spectrum

- The periodogram is an estimate of the spectral density of a signal. The term was coined by Arthur Schuster in 1898 (*the Schuster Periodogram*).
- A Power Density Spectrum is computed as the squared Fourier amplitudes with **some normalization**:

$$a_j = \sum_{k=0}^{N-1} x_k \, e^{2\pi i j k/N} \qquad j = -\frac{N}{2} + 1, \ldots, \frac{N}{2}$$

$$P_j = (Normalization)|a_j|^2$$

Power Spectrum – Leahy Normalization

- We will adopt the Leahy et al. (1983) normalization:

$$P_j = \frac{2}{N_{tot}}|a_j|^2 \quad j = 0, \ldots, \frac{N}{2}; \;\; where\ N_{tot} = N_{ph} = \sum_k x_k = a_0$$

N_{tot} – dispersion of the total number of counts in the time series. For the Poisson process, the variance (square of the standard deviation) is equal to the total number of counts.

Properties of Leahy normalized PDS

- Variance in the real time series x_k:

$$Var(x_k) \equiv \sum_k (x_k - \overline{x})^2 = \sum_k {x_k}^2 - \frac{1}{N}\left(\sum_k x_k\right)^2 =$$

$$= \frac{1}{N}\sum_j |a_j|^2 - \frac{1}{N}{a_0}^2 = \frac{1}{N}\sum_{j\neq 0} |a_j|^2$$

Parseval's theorem

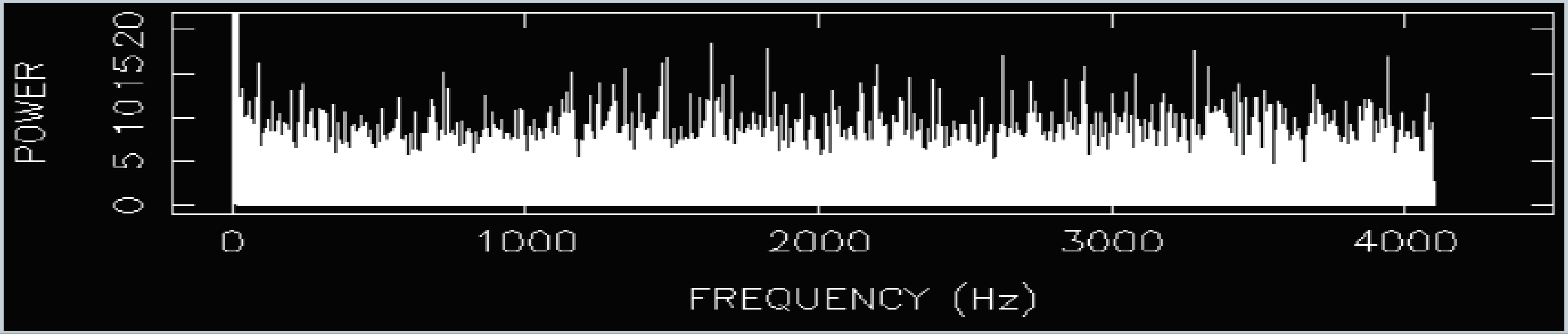
$$Var(x_k) = \frac{N_{tot}}{N}\left(\sum_{j=1}^{N/2-1} P_j + \frac{1}{2}P_{N/2}\right)$$

variance is sum of powers!

The dimension of P_j is the same as x_k and a_j: $[P_j] = [a_j] = [x_k]$

Properties of Leahy normalized PDS

- The Leahy normalization is chosen such that if the x_k are Poisson distributed, then the P_j exactly follow the chi-squared distribution with 2 dof, χ^2.
- Properties of this distribution:
  - The mean power is 2;
  - the standard deviation is 2!
- So, the power spectrum is very noisy. This does not improve with:
  - longer observation — you just get more powers
  - broader time bins — you just get a lower ν_{Ny}



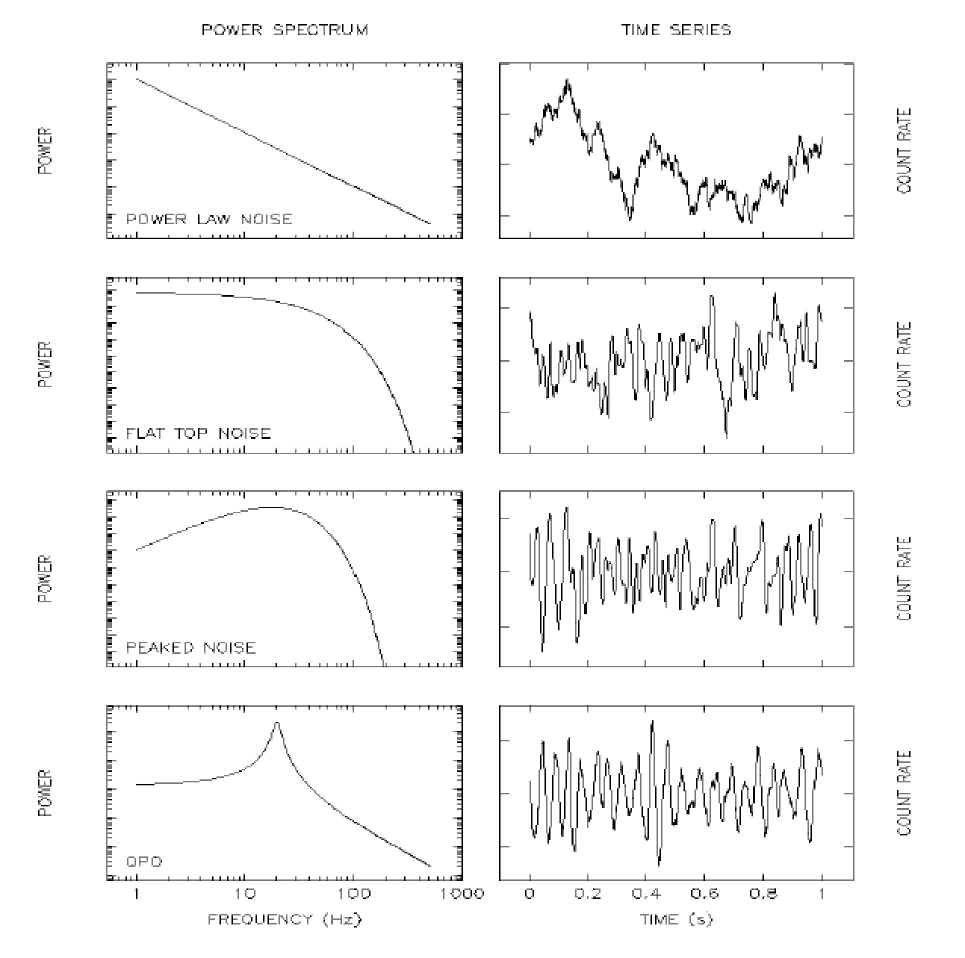
Statistics of Power Spectra

- Flux measurements are always accompanied by noise.
- The light curve can be divided into its independent components: the deterministic signal *S* and the noise *N*. For an individual time bin, the total number of counts is composed of the sum of the signal and the noise, i.e., $x_k = s_k + n_k$.
- Examples of deterministic signals:
  - a non-periodic deterministic variation, such as a nova light curve;
  - A periodic variation, such as an eclipsing binary or a RR Lyr light curve;
  - a multiply periodic variation, such as a spectroscopic triple system;
  - a modulated periodic variation where either the amplitude, frequency, or phase may vary with time - for example a pulsating system in a binary orbit.

Statistics of Power Spectra

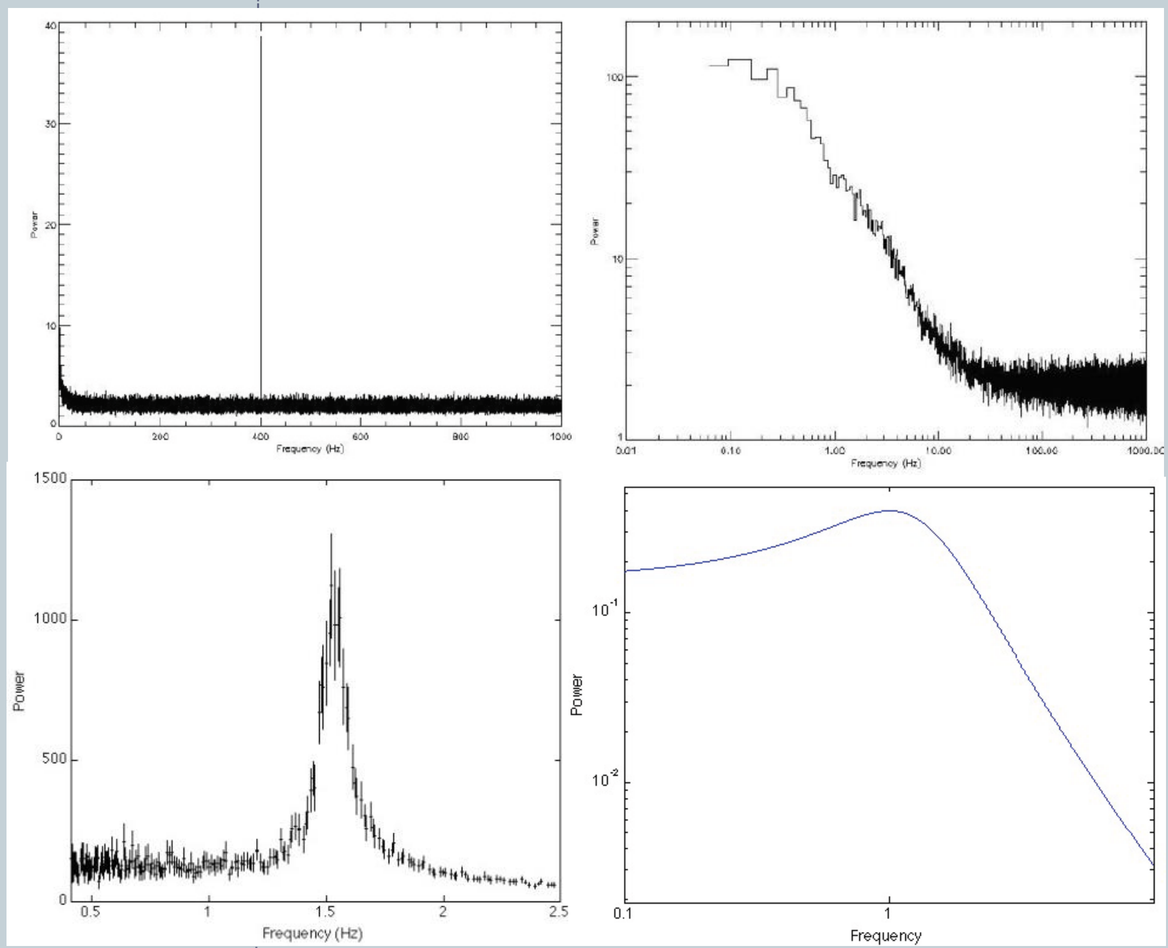
- 'Noise' (= random aka stochastic processes) in the light curve produces peaks and broad components in the power spectrum.
- Examples of noise:
  - Counting statistics noise (Poisson noise) -> white noise;
  - Poisson noise modified by instrumental effects (e.g. dead-time) and other instrumental noise;
  - Noise that is (stochastic) intrinsic source variability: QPO, band limited noise, red noise, etc.
- All these can occur at the same time, possibly together with deterministic signals.
- They can be the **background** against which you are trying to detect something else
- Or they can be the **signal** you are trying to detect.

Statistics of Power Spectra



Main types of signals

- Coherent pulsation
- Broad-band noise
- Broad peak (QPO)
- Peaked-noise



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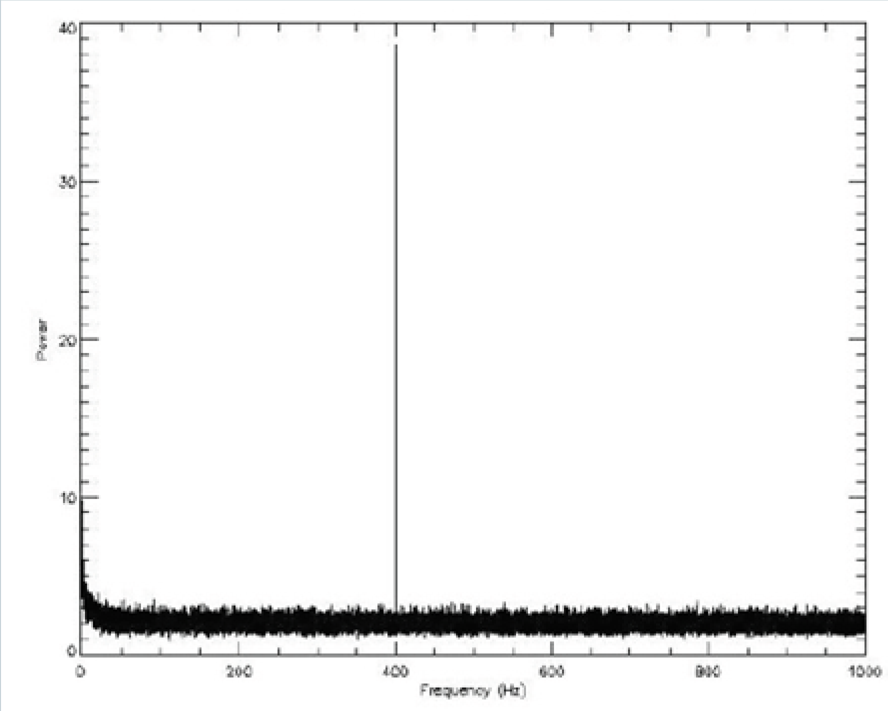

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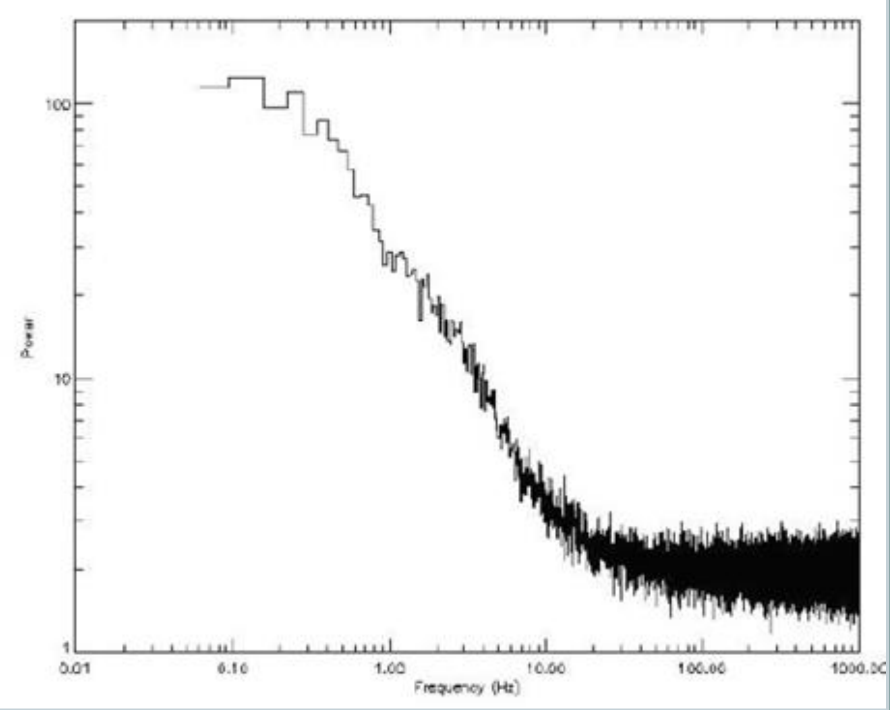

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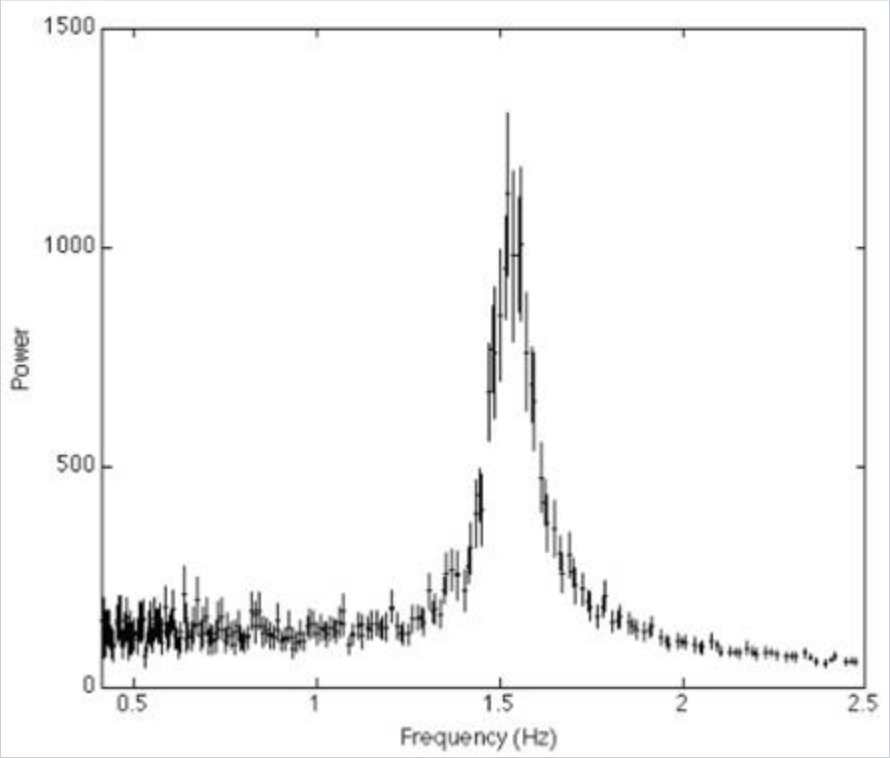

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